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Notes on How to Choose Parameter Values for the Cox Numerical Ocean Circulation Model

by

U.K. Met. Office Ocean Modellers; editor M.J. Bell

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Summary

This report is a summary of our knowledge about the parameters used by the Cox Numerical Ocean Circulation Model. Its aims are: (1) to ensure that long integrations of the ocean model are not performed with very poor choices of parameters; (2) to make it easier to understand various aspects of the ocean model.

Each section discusses a group of related model parameters. It

defines the parameters

cites helpful references

discusses the main factors influencing the choice of parameters

indicates the importance for the model of the parameters (e.g. sensitivity of results to parameter variation, symptoms of incorrect settings)

discusses the resolution dependence of the parameter choice

recommends parameter choices and ranges of acceptable values

It is intended that these notes be kept up-to-date as a working document. Suggestions for corrections or improvements will be very gratefully received.

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1. Parameters related to timestepping

1.1 Definition of parameters

DTTS timestep for tracers (e.g. temperature and salinity) (in seconds)

DTUV timestep for baroclinic (depth-varying) velocities (in seconds)

DTSF timestep for barotropic streamfunction calculation (in seconds)

RAT(Z) scaling factors for increase of tracer timestep with depth

NMIX number of timesteps between 'mixing timesteps'

PNU the constant γ for Robert time filtering

ACOR coefficient for implicit treatment of Coriolis term (α in Cox (1984).

The Cox model uses a leapfrog scheme in which twice the rate of change of the field F at timestep n (i.e. F_n) is added to the field at step n-1 to form F at step n+1:

$$F_{n+1}=\overline{F_{n-1}}+2\Delta t(\partial F/\partial t)_n \qquad (1.1)$$

The overbar in (1.1) is defined in the next paragraph. The factor Δt multiplying the time derivative in (1.1) at depth Z is DTTS*RAT(Z) for tracers, DTUV for the horizontal velocities, and DTSF for the streamfunction.

When the Robert filter is used, the field at step n is smoothed after F_{n+1} is calculated;

$$\overline{F_n}=F_n+\gamma(F_{n+1}-2F_n+\overline{F_{n-1}}) \qquad (1.2)$$

see GFDL Ocean Model Circular No. 4 and Haltiner & Williams 1980 (hereafter HW) section 5-7-3. The overbar indicates that the field has been smoothed in this way. This filter is applied to all fields (tracers, horizontal baroclinic velocities and streamfunction).

1.2 References

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1.3 Factors influencing choice of settings

As the amount of computation required to complete a run is proportional to the model timestep it is desirable to make the model timesteps as long as possible. However, if a long timestep is chosen the model may be numerically unstable and the solution blow up before the end of the integration. The longest time step which can be used depends on the processes and waves represented in the model and details of the numerical formulation. We consider the constraints due to advection at a constant velocity, external gravity waves, Rossby waves and inertial-gravity waves in turn. Constraints due to diffusive fluxes are described in sections 2 and 3.

The simple 1D advection equation:

$$\partial F/\partial t + c\partial F/\partial x = 0 \qquad \textbf{(1.3)}$$

solved using the leapfrog scheme is stable if the CFL condition:

$$c\ \Delta t < \Delta x \qquad \textbf{(1.4)}$$

is satisfied (HW section 5-3), Δt being the time step and Δx the grid spacing. On a 2D grid the condition on c Δt is $\Delta x / 2^{1/2}$ (HW section 5-6-7).

For equations of a less simple form the CFL condition is a useful guide to which waves are likely to give instabilities if Δx is interpeted as the quarter wavelength of a wave and c as its phase speed (or possibly group speed) for the continuous equations of motion. If (1.4) is not satisfied at any wavelength, those waves are likely to require a special formulation of the dynamical equations.

Short wavelength external gravity waves are thus expected to require very short timesteps when they are present as they have phase speeds of order of 200 m/s. The rigid lid condition is used in the standard model configuration to exclude these waves from the model. If the free surface option is used, the barotropic timestep must be chosen to resolve the surface waves (see section 7).

External Rossby waves do not provide similar constraints on the time step although the largest scale Rossby waves may have phase speeds of 100 m/s, because short waves move slowly. See appendix A for a simple illustration of this point for the vorticity equation on a β-plane.

In non-linear equations these CFL criteria need a little adjustment. The total velocity may need to be the sum of the tracer advection and phase speeds. Also in some representations of the momentum equations, instabilities arise due to aliasing of waves shorter than those represented by the grid into two to four grid point waves (HW section 5-11). This cannot happen in models which conserve both enstrophy and energy. The Cox model uses a flux formulation in order to conserve energy; it is not clear that it conserves enstrophy.

In models with coarse grids it is desirable to use timesteps of the order of a day. In order to do this internal inertial-gravity waves (with angular frequency ϖ greater than $f = 10^{-4}$ s^{-1}) of short periods compared to the timestep need to be removed. This can be done by treating the Coriolis terms in the momentum equations semi-implicitly; that is by replacing:

$$\begin{array}{llll} -fu_n & by & -f(\alpha u_{n+1}+(1-\alpha)u_{n-1}) \\ and\ fv_n & by & f(\alpha v_{n+1}+(1-\alpha)v_{n-1}) \end{array} \qquad (1.5)$$

As shown by the analysis of Killworth et al. (1984) and sketched in appendix A, for the shallow water equations on a B grid, the stability criterion is then of the CFL form when α = ACOR $\geq$ 0.5.

It is possible to reduce the amount of computer time required to run the model to equilibrium by multiplying the time derivative terms in the momentum equations by an artificial factor, DTTS/DTUV. This has the effect of reducing the frequency and slowing down the short internal inertial-gravity waves by a factor of $(DTTS/DTUV)^{1/2}$, (Bryan 1984). According to Killworth et al (1984) (again see appendix A) for a given value of tracer timestep with α =

ACOR = 0.5, the internal waves are stable (according to a von Neumann analysis) when:

$$DTTS/DTUV > \left(\frac{2\ c\ DTTS}{\Delta x}\right)^2 \qquad (1.6)$$

where c is the phase speed of the first baroclinic mode (about 3 m s^{-1}). In practice, one sets DTTS to be as large as possible, then sets DTUV to be as large as possible consistent with (1.6).

Use of too large a value for DTTS/DTUV would distort the long wavelength planetary waves.

The convergence of the deep ocean takes two orders of magnitude longer than that of the surface layers, necessitating long spin-up runs if the deep ocean is modelled. The computational time required to do this can be reduced by scaling the timestep used in the tracer equation by a coefficient, RAT, that increases with depth. This method takes advantage of the fact that a longer timestep can be employed in the deep ocean without violation of the CFL condition because the advective velocities are an order of magnitude smaller there. The use of RAT implies that the thermal capacity of each layer is altered, thus distorting the stratification and increasing the phase speed c for the first internal mode (Bryan 1984 p 671). The ratio DTTS/DTUV should then need to be increased accordingly. However, experience at both GFDL and the Hadley Centre (see Tim Johns) suggests that in practice it is possible to use a depth-dependent timestep without decreasing the value of DTUV.

Use of a leapfrog scheme for solution of the equations of motion has the disadvantage that it supports a non-physical solution of the equations, called the computational mode, which reverses sign between one timestep and the next. This computational mode is satisfactorily damped by using the Robert filter on each timestep with a small value for the coefficient γ(see HW section 5-7-3 for the response function of this filter). An alternative (and far less satisfactory) approach is to employ an Euler forward timestep (instead of a centred leapfrog one) every NMIX timesteps. In the UM such a forward timestep is taken each time a dump is output as the dump contains fields on only one time level and it has been decided that the integrations must be repeatable from the dumps produced. Frequent dumping will disrupt the model evolution and should be avoided.

1.4 Importance in model

Use of too large a timestep usually has an obvious effect in that the model rapidly "blows up" (ie. goes unstable), generating increasingly large temperatures and velocities, and eventually failing with a floating point exception. This is due to an explosive growth of the small errors inevitably present in the numerical approximations. When setting up a new model it is as well to experiment with changing the timestep in order to obtain the largest possible value.

Provided that the timestep is chosen so as to keep the model stable, model results should be (at least it is desirable that they should be) independent of timestep used. However, there does appear to be some timestep sensitivity in the ice, mixed layer and isopycnal diffusion schemes, particularly when a timestep of order 1 day is used.

In accelerated integrations the ratio DTTS/DTUV should be chosen carefully as too small a value for DTUV could degrade the large scale planetary waves.

1.5 Resolution dependence

The timestep required to satisfy the CFL criteria decreases linearly with increasing horizontal resolution. The maximum value of timestep given by diffusivity considerations decreases with Δx^2 (see section 3).

1.6 Recommended / acceptable values

Most 1° models used at the Met Office have synchronous time-steps (ie DTTS=DTUV=DTSF) of 1800 seconds (i.e. half an hour), although the quasi-1.25° model and FOAM global 1° x 1° models were able to use a timestep twice as long as this, and the tropical model (which does not extend beyond the sub-tropics and omits the barotropic streamfunction) ran with a timestep of 4320 seconds.

Asynchronous, or accelerated, timestepping has been used with the 2.5° x 3.75° grid. The following settings gave acceptable results for a seasonal run set up by Richard Wood with standard climate settings of DZ:

DTTS = 86400 DTSF = 3600 DTUV = 3600
RAT = 13*1.0 , 1.51 , 2.13 , 2.75 , 3.38 , 4.0 , 4.0 , 4.0

For annual-mean-forced runs I have used larger values of DTTS/ DTUV:

DTTS = 172800 DTSF = 120 DTUV = 120
RAT = 12*1.0 , 2.96 , 4.42 , 6.22 , 5*8.0

PNU is usually set to be 0.01 in which case NMIX should be set equal to the dumping period. We do not recommend running with PNU set to zero.

ACOR has been set to 0.75 in all Met. Office integrations: ACOR = 0.5 may be a better choice.

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2. Parameters for vertical mixing

2.1 Definitions of Parameters

FKPM_SI Vertical viscosity (momentum diffusivity) (P&P scheme not selected)

FKPH_SI Vertical tracer diffusivity (P&P scheme not selected)

FNUB_SI Background viscosity (P&P)

FNU0_SI Coefficient for Ri-dependent part of viscosity (P&P)

FKAPB_SI Background tracer diffusivity (P&P)

STABLM_SI Maximum allowed diffusivity (momentum and tracers) (P&P)

GNUMINC_SI Minimum allowed viscosity (momentum diffusivity) between top two levels (P&P). The diffusivity at the sea surface is also set to this value.

GNUMINT_SI As GNUMINC_SI, but for the tracer diffusivity (P&P).

Units for all variables are m^2s^{-1}. P&P refers to the Pacanowski and Philander 'K-theory' mixing scheme, which is an option in the UM. In this scheme the vertical diffusivities for momentum and tracers, K_m and K_t are given by

$$K_m = \frac{FNU0_SI}{(1 + 5Ri)^2} + FNUB_SI \qquad (2.1)$$

$$K_t = \frac{K_m}{(1+5Ri)} + FKAPB_SI \qquad (2.2)$$

where Ri is a local Richardson number. The scheme is described in more detail in Ineson and Gordon 1989 and in Pacanowski and Philander 1981.

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2.3 Factors Influencing Choice of Settings

When the mixed layer model is present, the viscosity and diffusivity typically take their maximum values FNU0_SI+FNUB_SI and FNU0_SI+FNUB_SI+FKAPB_SI throughout the mixed layer, and revert to their background values FNUB_SI and FKAPB_SI below the mixed layer; it is only at the base of the mixed layer that intermediate values are taken. The mixed layer physics is thought to swamp the effect of vertical tracer diffusion within the mixed layer. However since the mixed layer model does not mix momentum, momentum

diffusion is important. Typically the model produces a momentum boundary layer (Ekman layer) which is thinner than the mixed layer; it is widely assumed that this happens in the real ocean too, in which case the difference in the two boundary layer thicknesses may be important in determining the subduction of mixed layer water into the permanent thermocline (Marshall et al., 1993). However recent observational work (Chereskin and Roemmich, 1991) suggests that in some regions of the ocean the Ekman layer might be DEEPER than the temperature mixed layer.

STABLM_SI is a maximum value of the vertical diffusivity, which is imposed for reasons of numerical stability if an explicit vertical diffusion scheme is used. However the implicit vertical diffusion scheme is unconditionally stable, and so this parameter should be redundant when the implicit scheme is chosen.

2.4 Importance in Model

The vertical MOMENTUM diffusivity (viscosity) controls the detailed profile of the surface Ekman layer (bottom Ekman layers are normally unresolved in GCMs), and plays an important role in determining the vertical structure of the high shear zone associated with the equatorial undercurrent. Results of Bryan (1987) suggest that otherwise the large scale flow is not very sensitive to this quantity, in coarse-resolution models. Note that with the parameter settings described in 2.6, the surface Ekman layer is not resolved, or only barely resolved, in the GCMs which we run; however since the integrated transport of the Ekman layer is (wind stress)/(density * f), which is independent of the viscosity and Ekman layer thickness, we can still expect to get the correct transport (and hence the correct Ekman pumping, which is what drives the large scale circulation). However in the case where the Ekman layer is deeper than the mixed layer (Chereskin and Roemmich, 1991), it is likely that the Ekman depth can influence the meridional heat transport. Carolyn Roberts is currently (September 1994) looking into this question.

The vertical TRACER diffusivity, however, plays a key role in the structure of the thermocline and in the thermohaline circulation. In practice FKPH_SI or FKAPB_SI is the key parameter (see 2.3 above). For large values ($>10^{-5}$ m^2s^{-1} or so), increasing the value leads to a thicker thermocline, a stronger overturning (NADW) cell, and stronger meridional heat transport. For values $<10^{-5}$ m^2s^{-1} , the thermocline thickness is determined by the wind stress and mixed layer processes; however deep water mass properties are still controlled by the diffusion in this case. See Bryan 1987 for a discussion of this in a sector model. Richard Wood has experience with the global GCM.

2.5 Resolution Dependence

No studies are known in this area. As horizontal resolution becomes finer one can turn down the horizontal viscosity, and there is an article of faith that for a given vertical diffusivity (and hence meridional density gradient) one will therefore get stronger overturning and heat transport (see Bryan 1987). However this does not appear to be the case in the results of Covey 1992. Experiments by Malcolm Roberts with a 1° x 1° North Atlantic model suggest that the sensitivity of the circulation to vertical diffusivity may be less at this resolution than at coarse climate resolution (2.3° x 3.75°), but this is currently only speculative.

2.6 Recommended Values and Range of Acceptable Values

The following values for the P&P scheme were chosen by Pacanowski and Philander (1981; P & P) and by Ineson and Gordon (1989; I&G) to optimise the equatorial undercurrent in their tropical Pacific models. Values used in the current global climate model are also shown.

TABLE 2.1 Parameter values used in current GCMs; all values are in SI units ($m^2\ s^{-1}$)

	P&P	I&G	Global
FNUB_SI	$1.0\ 10^{-4}$	$5.0\ 10^{-5}$	$1.0\ 10^{-4}$
FNU0_SI	$5.5\ 10^{-3}$	$5.5\ 10^{-3}$	$5.5\ 10^{-3}$
FKAPB_SI	$1.0\ 10^{-5}$	$5.0\ 10^{-6}$	See below
STABLM_SI	Not known	$1.0\ 10^{-2}$	$1.0\ 10^{-2}$ (but see 2.3)

It is not known how appropriate these values are (or indeed how appropriate the whole scheme is) for regions other than the Pacific equatorial undercurrent. However Ineson and Gordon found that the P&P scheme on its own (with the above parameters) produced mixed layers in the tropical Pacific which were too warm and shallow, especially in rapidly shallowing situations. It was for this reason that the Kraus-Turner mixed layer model was introduced, to run alongside the P&P scheme (see sections 2.3 and 7 (mixed layer)).

Observational evidence suggests that a value of FNU0_SI an order of magnitude larger than

the one here is appropriate for the surface Ekman layer (Chris Gordon, pers. comm.); using the value quoted gives an Ekman layer thickness Sqrt(2*FNU0_SI/f) of order 10 m at 45°N, whereas a thickness of order 50 m would be more appropriate. Thus the Ekman transport in our models tends to be trapped too near the surface. See section 2.4 for further discussion of this.

As described above, FKAPB_SI is a crucial parameter in determining the thermohaline circulation and deep water mass properties, and the values used in the tropical models are generally considered too low for the deep ocean. Kraus (1990) gives an estimate of this parameter from indirect, semi-observational methods, which ranges from $O(10^{-5}\ m^2s^{-1})$ at the surface to $O(1.5 \times 10^{-4}\ m^2s^{-1})$ at the bottom, and the only attempt to measure the value directly confirms that $1.0 - 1.5 \times 10^{-3}$ is an appropriate value for the upper 1000m or so (Ledwell et al 1993). Accordingly, the current global GCM uses a value for FKAPB_SI which varies linearly with depth, taking values 10^{-5} at the surface and 1.5×10^{-4} at 5000m. An update is required to implement this at UM version 3.2, but the depth-dependent formulation is available through the UI from version 3.3 onwards. Radiocarbon simulations (Taylor 1994 and in prep.) suggest that using the increasing value with depth can produce appropriate ages for the deep water masses without making the surface layers over-diffusive.

The parameters GNUMINC_SI and GNUMINT_SI are currently hardwired into the code in subroutine SET_CONSTANTS_OCEAN at values of $10^{-3}\ m^2s^{-1}$. These parameters are a fix to prevent isolation of the top level when using DZ =10 m. A different value may be appropriate if using a different surface DZ, but there is no experience in this area.

When not using the P&P scheme, the same considerations apply to the choice of FKPH_SI as to FKAPB_SI above. Global models (which usually have 50m top layers) are not thought to be very sensitive to the choice of FKPM_SI, and values between 10^{-4} and $5.0 \times 10^{-3}\ m^2s^{-1}$ are found in the literature.

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3. Parameters for horizontal mixing

3.1 Definitions of parameters

There are two options: Isopycnal diffusion and non-isopycnal diffusion. The former diffuses tracers along isopycnals (constant density surfaces), which is what is mainly thought to occur in nature. To do this, a diffusion tensor has to be rotated from isopycnal coordinates to our model frame of reference, as shown by Redi (1982) and implemented by Cox (1987). Non-isopycnal diffusion is simply horizontal diffusion of tracers.

There is a modification to our isopycnal diffusion scheme which does not give a diffusive flux across the isopycnals due to limiting the mixing slope (see Gerdes et al. 1991, and Roberts et al. 1994, the former for the theory, the latter for some results of using it). There is also the Gent and McWilliams (1990, 1994) isopycnal-thickness diffusion scheme, which is not yet implemented in our model.

The parameters for the isopycnal and non-isopycnal diffusion schemes are now explained:

Isopycnal diffusion:

AI is the 'along isopycnal' diffusion coefficient for tracers, it appears in the diffusivity tensor which gives the coefficients of diffusion resolved along isopycnal surfaces (see UM doc no. 51). It is allowed to vary with depth, z:

$$AI = AHI2_SI + (AHI1_SI - AHI2_SI)\exp(-z/AHI3_SI) \qquad (3.1)$$

AHI1_SI surface 'along isopycnal' diffusion coefficient of tracers

AHI2_SI deep 'along isopycnal' diffusion coefficient of tracers

AHI3_SI scale depth, a measure of the rate at which the diffusion coefficient varies with depth.

AH_SI coefficient of horizontal diffusion of tracers, which acts as a background diffusion to smooth out the tracer fields.

SLOPE_MAX is the maximum allowable slope of isopycnals for diffusion to ensure numerical stability

AM_SI coefficient of horizontal diffusion of u,v, optionally takes latitude-dependent form AM0_SI + AM1_SI*cos(latitude) (see 3.3.1.3).

Non-isopycnal diffusion:

AH_SI coefficient of horizontal diffusion of tracers.

AM_SI coefficient of horizontal diffusion of u,v, optionally takes latitude-dependent form: AM0_SI + AM1_SI*cos(latitude) (see 3.3).

Units: AHI1_SI, AHI2_SI in m^2s^{-1}, AHI3_SI in m, SLOPE_MAX non-dimensional, AM_SI in m^2s^{-1}, AH_SI in m^2s^{-1}.

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UM Documentation paper No.51

3.3 Factors influencing choice of setting

3.3.1 Restrictions on momentum diffusivity AM_SI

There are four such restrictions, all of which must be satisfied. The first two involve the grid spacing only, the latter two involve the timestep as well.

3.3.1.1 Frictional western boundary layer

AM_SI needs to be large enough for the model to resolve western boundary currents (wbc), because if they are not resolved a computational mode may be excited which extends out to the interior, giving a checkerboard pattern to the streamfunction and possibly resulting in a model blowup.

The width of the boundary layer is dependent on the boundary layer type, which is determined from the dominant terms in the momentum equations (see Gill p518). Given a fixed resolution model (a fixed grid spacing), a lower bound can then be imposed on AM_SI to give a boundary layer width that is resolvable.

For a diffusive western boundary layer (this type is the one encountered at currently used resolutions), in which lateral friction and the pressure gradients balance the velocities, the criterion to be satisfied is (Bryan 1975 and Gill):

$$AM_SI \succ \beta(\Delta x\, 3^{1/2} / \pi)^3 \qquad (3.2)$$

where beta= $\beta = 2.28\ 10^{-11}$ cos(latitude) $m^{-1}\ s^{-1}$ and Δx is the grid spacing.

3.3.1.2 Grid Reynolds number

Another condition on AM_SI arises when using centred differencing (which our model does). A computational mode may again be excited, and this would give a solution which would oscillate with sign over adjacent grid points (text and appendix B of Bryan 1975, and Roache 1976 pp 161-165). To prevent this, the grid Reynolds number needs to be less than 2:

$$Re(grid) < 2 \Rightarrow AM_SI > U\,\Delta x / 2 \qquad (3.3)$$

(where U is a typical horizontal velocity scale)

In the literature, horizontal velocity scales of 10 cm/s are used with this criterion, which are representative of typical advective velocities in models (see Bryan 1975, Mead 1989 for more details).

For coarse resolution models ($\Delta x > 1^\circ \Rightarrow$ AM_SI > 5500 m^2s^{-1}), 3.3.1.1 is the more restrictive of the above two criteria, whereas for fine resolution ($\Delta x < 1^\circ$) 3.3.1.2 becomes the more restrictive (see Mead thesis, Fig. 2.2).

3.3.1.3 Stability of the diffusion scheme

Given a value for AM_SI, the analysis of the simple diffusion equation when solved using the leap-frog (centred-differencing) scheme presented in appendix B yields a condition on the timestep:

$$\Delta t \leq \frac{\min(\Delta x^2, \Delta y^2)}{4\,AM_SI} \qquad (3.4)$$

With the convergence of parallels at high latitudes, this put a severe constraint on the timestep in the 2.5° x 3.75° model, leading to the use of a latitude-dependent AM_SI:

$$AM_SI = AM_SI(0) + AM_SI(1)\cos(latitude) \qquad (3.5)$$

so that AM_SI = AM_SI(0) + AM_SI(1) at the equator, and AM_SI = AM_SI(0) at the pole (this is purely for computational reasons, not scientific ones).

3.3.1.4 Topographic Instability

There is also the Killworth instability, which comes about due to steep topography and the

convergence of parallels at high latitudes (Killworth 1987). In regions of steep topography, with depths H1,H2 at adjacent gridpoints and H2>>H1, the stability criterion is:

$$\Delta t < \frac{2\,H1\,\Delta x^2}{H2\,AM_SI} \qquad (3.6)$$

This may then be more severe than the constraint (3.4) above and in fact is what currently limits the timestep in the 2.5° x 3.75° model.

3.3.2 Restrictions on tracer diffusivity AH_SI, AHI1_SI etc

3.3.2.1 Grid Peclet number

There is a similar restriction for AH_SI, with a grid Peclet number, as there is for AM_SI with a grid Reynolds number (see above), but this does not restrict AH_SI because of strong geostropic coupling between velocity and density fields. This means that the lateral viscosity tends to damp out computational modes and values of AH_SI can be much lower than AM_SI (Bryan 1975).

3.3.2.2 Isopycnal mixing slope restriction

A constraint on the isopycnal diffusion arises from a stability analysis of the simple diffusion equation when solved using the leap-frog (centred-differencing) scheme (see Cox 1987). The most restrictive constraint is alleviated by solving the "zz" component of the diffusion equation implicitly, leaving the next most severe constraint:

$$SLOPE_MAX \leq \frac{\min(\Delta x,\Delta y)\Delta z}{\Delta t\,AI} \qquad (3.7)$$

which allows slopes of 0.01 (typical value in wbc) (see comment in IPDCOFCL). In most of the ocean slopes do not exceed this value, but there are areas of higher gradients (such as regions of dense overflow like that from the Greenland-Iceland-Norway basin into the N. Atlantic) in which these slopes can be exceeded. In these areas, the standard scheme causes an enhancement of the cross-isopycnal diffusion, leading to a dilution of the properties of the

overflow (see Roberts et al 1994b).

However, even if (3.7) is satisified by more than a factor of 10, noise can still result from the isopycnal diffusion (see Bell (1994) for some results). Appendix B shows that the off-diagonal mixing term generates 4 gridpoint wavelength noise in the x-z plane which is not damped out by the horizontal diffusion unless

$$|R| > 2 \;\; ; \;\; where \; R = \frac{-\rho_x/\rho_y}{\Delta z/\Delta x} \qquad (3.8)$$

This condition says that an isopycnal must not slope so steeply that it changes height in the vertical by two or more levels between neighbouring horizontal grid points. Hence for a model with 100 metre thick layers in the thermocline and 100 km horizontal resolution, SLOPE_MAX should not be larger than about 0.002 (the experiments in Bell (1994) used SLOPE_MAX = 0.01). Fortunately this instability evidently stops growing when it reaches finite amplitude and starts to disrupt the slope of the isotherms.

It is reasonable to expect, and appears to be the case in practice, that the amplitude reached by the isopycnal noise can be reduced by decreasing AH_SI. The scheme described in Gerdes et al (1991) does this, scaling the isopycnal diffusion according to the slope of the isopycnals. However, when this scheme was compared to the standard scheme (Roberts again) it was found that convection took up much the same role as diffusion had previously, and there was little overall change in the water properties downstream of the overflow.

3.4 Importance in model

Larger values of AM_SI will tend to broaden the wbc (as width of diffusive wbc is proportional to (AM_SI/beta)**1/3). Low values of AM_SI will lead to noisy streamfunction and velocity fields (perhaps terminal), a symptom of which could be high model energies. Too high values may decrease the maximum possible timestep (see 3.3).

Given a value of AM_SI, there are two constraints on the timestep. The Killworth instability results in a +/- blowup in the streamfunction near steep topographic slopes at high latitudes, typically near the Antarctic shelf edge. This is what currently limits the timestep in the global 2.5x3.75 global model. At high latitudes (typically the Arctic Ocean) a diffusion condition limits the timestep (see 3.3.1.3).

Too low values of AH_SI (or equivalent AI) will lead to noisy tracer fields (especially in the

lower thermocline), and hence possibly noisy velocities (again, perhaps terminal). If AI is chosen small, a larger value of AH_SI may smooth out most of the noise (but will be rather less realistic).

3.5 Resolution dependence

Values for AM_SI used in 1x1 degree models still need to be higher than 'observations' imply, in order to keep the models stable. For coarser resolutions, the values are highly resolution dependent (see 3.3)

Values of AM_SI in the ocean interior are not widely quoted, and often not derived from model studies at eddy resolution because of large spatial variability (the values would be negative in regions of significant eddy activity, demonstrating that representation of momentum transport by diffusion is inappropriate in these areas). However, for models of 1x1 degree resolution or coarser, simple eddy viscosities have to be used for numerical reasons, and values of O(1000 m^2s^{-1}) are thought to be 'reasonable' (see Semtner and Holland 1980 for value of 2000 m^2s^{-1} derived from high resolution idealised model, other refs. have values of O(300 m^2s^{-1}) but these are not necessarily applicable to models of our resolution).

AI is also very spatially variable, and there are similar problems to those for AM_SI. However, for the reason mentioned in 3.3, the values are not so constrained by stability considerations, and observational values can be attained. These seem to indicate values of O(1000 m^2s^{-1}) at the surface, with smaller values at depth, and modelling studies seem to give the same order of magnitude (see eg. Bryan & Lewis, Garrett and many others for observational references). Many observations are taken in rather atypical areas of the ocean (often looking at eddies or other things) so some values may be rather unrepresentative.

In early models that used only horizontal, as opposed to isopycnal, diffusion of tracers it was found that the relative sizes of AM_SI and AH_SI were important, and that if AH_SI was too large, too much upwelling would occur in the western boundary current. This is still relevant with isopycnal diffusion at our resolutions, and our models still have too much upwelling. The new Gent and McWilliams scheme (not yet implemented) is claimed to fix this. It is still advisable to keep the value of AH_SI as low as possible (for the relevant resolution), to minimise the upwelling (see Holland 1989).

3.6 Recommended values and range of acceptable values

(these are for near 1° x 1° resolution, see end for 2.5° x 3.75° examples)

1000 m^2s^{-1} <= AHI1_SI <= 5000 m^2s^{-1} (dependent on which observations you look at and where they are taken, but 2000 m^2s^{-1} seem to be a favoured value)

200 m^2s^{-1} <= AHI2_SI <= 500 m^2s^{-1} (500 is widely used, set AHI1_SI = AHI2_SI = desired value for no vertical variation), AHI3_SI = 500 m is mainly used

20 m^2s^{-1} <= AH_SI <= 80 m^2s^{-1}, if using isopycnal diffusion, this leaves some +/- noise in the lower thermocline, which is manageable. Most centres (eg GFDL) have used values like 750, though generally with resolutions coarser than 1° x 1° (eg England 1993). Using AH_SI=40 m^2s^{-1} also results in wavelike variability (timescale O(decades)) in the ACC, when long (1 day) timesteps are used with distorted physics. This results in a drift when switching to normal timesteps. Setting AH_SI = 400 m^2s^{-1} damps out the variability and fixes the drift.

If isopycnal diffusion is not used then AH_SI=horizontal diffusion of tracers, see range for AHI1_SI above

SLOPE_MAX = 0.01 is usually sufficient, a reasonable slope in a western boundary current, though (3.8) suggests that to avoid noisy thermal fields in 1° x 1° models with 100 metre thick layers in the thermocline one should take SLOPE_MAX nearer to 0.002.

4500 m^2s^{-1} <= AM_SI <= 10000 m^2s^{-1} (more usually around 6000, 4500 near stability limit for 1° x 1° resolution, value rather dependent on global/limited area model used and resolution, eg TOGA model uses 10000, N.Atlantic 1° x 1° model uses 2000, but is quite noisy). When using the latitude-dependent form, we have so far always set AM_SI(0)=AM_SI(1).

For reference, in the global 2.5° x 3.75° model: AM_SI(0)=AM_SI(1)=1.5 10^5 m^2s^{-1}, AHI1_SI=2000 m^2s^{-1}, AHI2_SI=400 m^2s^{-1}, AHI3_SI=500m, AH_SI=40 m^2s^{-1} (but see note above).

The GFDL model, as described in England 1993, is 3.75x4.5 degree and uses: AM_SI=2.5 10^5 m^2s^{-1} , AHI1_SI=5000 m^2s^{-1}, AHI2_SI=1000 m^2s^{-1}, AHI3_SI=500m, and a horizontal tracer diffusion which varies in the vertical in the same way as the isopycnal diffusion does (see 3.1), with surface AH_SI=750 and bottom AH_SI=375. This value for the horizontal diffusion is much larger than the values we use, and would seem to devalue the use of isopycnal diffusion somewhat. However, having a substantial component of diffusion along

the isopycnal surfaces is still important in large-scale water mass formation (particularly AAIW - see England 1993).

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4. Parameters for relaxation solution for streamfunction

4.1 Definitions of parameters

MXSCAN Maximum number of iterations in a timestep for relaxation scheme that calculates streamfunction.

SOR Relaxation coefficient (amount of increment to streamfunction added to developing solution each iteration).

CRIT Convergence criterion; iteration stops when the ratio of the {maximum increment to the streamfunction tendency in an iteration} to {the maximum streamfunction tendency} is less than CRIT.

4.2 Important references

Unified Model documentation paper 40: the Ocean Model.

Smith, G.D. 1969 Numerical solution of partial differential equations. Oxford University Press.

4.3 Factors influencing choice of settings

These parameters are only applicable if the model is being run with the streamfunction form of the barotropic solution. Thus some versions of the tropical model and runs using the free surface formulation do not need to specify these parameters. They affect the calculations by subroutine RELAX

Note that although the Met. Office ocean model is closely related to the original version by Cox, the convergence criteria (how CRIT is interpreted) differ from Cox's approach, and also from any of those used in the MOM.

To derive the streamfunction from the tendency of barotropic vorticity requires a Poisson equation to be solved. **This equation is for the change in stream function (tendency times time step)**; the resulting value is used to calculate the streamfunction at the next time level.

This has the numerical advantage that the magnitude of the stream function tendency varies less than the stream function itself. the equation solved is described in UMDP 40.

The original version of the model used successive over relaxation. In this the latest values of the streamfunction are used to update the next point - a process that is scalar. To vectorise the calculations and make them parallel a chequerboard approach can be chosen. This may slow convergence (ie needs more iterations) but may take less computer time (because the code can vectorise and run in parallel on different processors). Semtner uses a scheme that has even slower convergence but that runs effectively over many processors.

The optimal value of **SOR** can, in theory, be calculated once the governing equation is known. It depends on the eigenvalues of the matrix representing the finite difference calculations. In practice this is not possible, and we have to rely on experimentation and guided guesswork.

Smith (1969) gives the formula $\omega = 2 - \sqrt{2}\pi(\frac{1}{p^2} + \frac{1}{q^2})$ for the optimum value of SOR for the Poisson equation in a rectangular domain (using full successive over-relaxation, not the chequer-board approach). Here p and q are the number of points in each grid direction. Strictly, this formula does not apply directly to the calculations in RELAX, but it can be used to guide the choice of SOR. Thus SOR must increase as the number of points in the model increases. Also $0 < SOR < 2$.

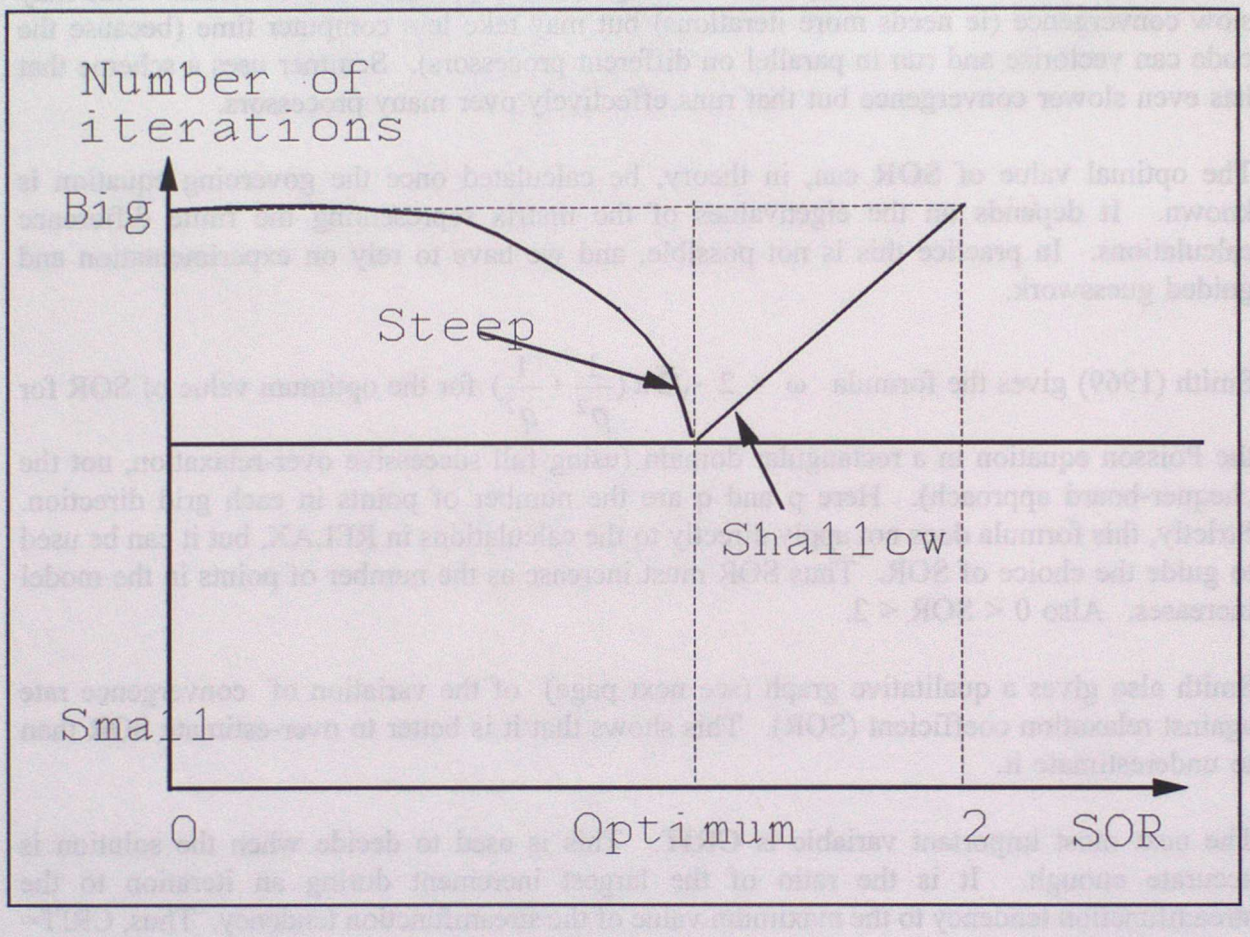
Smith also gives a qualitative graph (see next page) of the variation of convergence rate against relaxation coefficient (SOR). This shows that it is better to over-estimate SOR than to underestimate it.

The next most important variable is **CRIT**. This is used to decide when the solution is accurate enough. It is the ratio of the largest increment during an iteration to the streamfunction tendency to the maximum value of the streamfunction tendency. Thus, CRIT= 10^{-6} would represent 6 digit accuracy (compared to the largest value of the streamfunction change; this is comparable the precision of a 32 bit calculation). The smaller CRIT is, the more iterations will be needed but the solution will be more accurate. Remember that maximum values of streamfunction in a global model are around Antarctica and exceed 130Sv, while in coarse resolution models the Gulf Stream transport is less than 30Sv.

MXSCAN is the final parameter. It sets an upper limit on the number of iterations for the relaxation scheme. If the model run uses the maximum number of scans at most timesteps, the solution for the streamfunction will be poor. Often MXSCAN iterations are attained only at timesteps when the forcing is updated, and there it serves as a useful limit to the amount of computation. More iterations are needed for models with large numbers of gridpoints than for models with fewer points.

Setting any of these parameters to inappropriate values does not produce catastrophic results, it just degrades the model solution. Beware of a model that frequently uses MXSCAN

iterations; SOR might be inappropriate, CRIT too tight (remember that currents are only stored to 32 bit accuracy), or MXSCAN set too low.



4.4 Resolution dependence

All parameters are set subjectively, Experiments on short runs might indicate the sensitivity of the relaxation scheme to the values chosen. Higher resolution models have more gridpoints (and more complex geometry) than low resolution ones; larger values of SOR and MXSCAN should be expected for these. CRIT might be lower for a run with quasi-uniform stream function than for a global integration.

4.5 Recommended values

SOR: between 0 and 2. Experience shows 1.65 to be a reasonable starting point. Err on the high side.

CRIT: 10^{-4}; smaller values improve accuarcy but increase the number of scans needed.

MXSCAN: typically 100 to 300.

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5: Parameters for solar radiation

5.1 Definitions and parameters

The absorption by the ocean of the short wave part of the incoming solar depends upon the wavelength of the radiation - red light being absorbed more quickly than the blue end of the light spectrum. The classic way to treat this absorption is to divide the incoming radiation into a number of wave bands and specify an absorption coefficient for each wave band. The total absorption is then obtained by summing over all of the bands. A complex light absorption model of this type has been used by Woods et al (1984). In the unified model a simple two band approximation is used:

$$S(z) = S(0)\{Re^{\eta_1 z}+(1-R)e^{\eta_2 z}\} \qquad \textbf{(5.1)}$$

where S(z) is the short wave flux at depth z and S(0) the value of this flux at the ocean surface. R is the fractional distribution between the two bands and η_1 and η_2 are the absorption coefficients in each of the bands. The coefficients R, η_1 and η_2 are functions of water type and are given by Paulson and Simpson (1977) for the different types. In the code, R=RSOL, η_1=ETA1, η_2=ETA2.

5.2 Important References

Jerlov, N.G., (1976) Marine Optics: Elsevier Oceanogr. Ser., Vol.14, 231p.p., Elsevier, New York.

Paulson, C.A. and J.J. Simpson, (1977) Irradiance Measurements in the Upper Ocean. J. Phys. Oceanogr. 7, 952-956.

Simonot, J.Y. and H. Le Treut, (1986) A climatological field of mean optical properties of the world ocean. J. Geophy. Res., 91, No. C5, 6642-6646.

Woods, J.D., W. Barkman and A. Horch, (1984) Solar heating of the oceans, diurnal, seasonal and meridional variation. Q. J. R. Meteorol. Soc.,110, 633-656.

5.3 Factors influencing choice of settings

In practice the opacity of the ocean water varies both geographically and in time. It principally depends upon biological production in open ocean regions (in coastal waters murkiness from river outflow can be important). At any location, however, a certain water type predominates and a global dataset of climatological water type has been compiled by Simonot and Le Treut (1986). At the current stage of model development it suffices to use a constant value throughout the whole model. In choosing the water type an average type for the ocean domain should be used. In future, interactive biology models, already under development, will predict the value of these absorption parameters.

5.4 Importance in model

These parameters will affect mixed layer depth and SST. Maximum errors of approximately 1C can be expected in local areas. For example, if parameters appropriate to an open ocean water type are used throughout the model then this will underestimate the short wave absorption in coastal zones of strong primary production.

5.5 Resolution dependence

The double exponential formulation is not, in itself, resolution dependent. However, most of the radiation is absorbed in the top 10m or so and , therefore, if the top model level is thicker than 10m it is a waste of time to calculate both exponentials. It is better to parametrize the longer wavelengths by removing a constant fraction from S(0) and use a single exponential to decay the remainder. If this is done, allowance should also be made for implicit potential energy changes between absorbing the heating uniformly throughout the top layer and as an exponential function.

5.6 Recommended values and range of acceptable values

Refer to the global chart in Simonot and Le Treut (1986) which is reproduced below and determine an average water type. Paulson and Simpson then give the corresponding values of RSOL, ETA1 and ETA2. For the global ocean we currently use ocean water type IB (check) for which RSOL=0.67, ETA1=1 m^{-1}, ETA2=1/17 m^{-1}.

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6: Parameters for Haney forcing

6.1 Definitions of parameters

Haney forcing is a particular means of providing the surface heat fluxes for forcing ocean models and was introduced by Haney (1971). The latent, sensible and net long wave flux at the ocean surface are all functions of atmospheric variables (air temperature, wind speed etc.) and also of SST. If the atmospheric variables are held fixed then there is a simple negative feedback between SST and the surface heat flux i.e. warmer SST leads to greater evaporation, sensible heat loss and net long wave loss and therefore cools the surface. Haney's idea was to linearise these three heating components and express them all as functions of the air-sea temperature difference. So:

$$Q_{TOT} = Q_L + Q_S + Q_{IR} = \lambda(T_{air} - T_{sea}) \qquad (6.1)$$

where Q_{TOT} is the total heat loss (not including short wave effects) and the three terms in the centre are the latent, sensible and net long-wave losses respectively. is the, so called, Haney coefficient and on the right hand side is the air-sea temperature difference. λ can be calculated from the bulk formulae for the turbulent heat losses and the radiation formulae for the net I.R. using the assumption of a fixed air-temperature and climatological winds.

(*It is important to note that there is little physical justification for the use of Haney forcing. In practice, the air temperature tends to follow the SST rather than the other way around. In fact, calculations by Steve Foreman using a coupled model to calculate the geographical distribution of* λ *gave values in the range -100 to +100* $Wm^{-2}K^{-1}$. *A more physically reasonable approximation, often used in tropical models, is to assume the air-sea temperature difference is fixed and apply this directly into the bulk formulae (Philander and Seigel, 1985). Unfortunately this is not a good approximation outside of the tropics.*)

The total heating at the ocean surface includes the shortwave radiative heating. Haney introduced an 'effective air temperature' so that this flux could also be incorporated into the formulation. With S as the short wave heating, and Q_{net} as the total flux:

$$Q_{net} = S + Q_L + Q_S + Q_{IR} = S + \lambda(T_{air} - T_{sea}) = \lambda(T_{air}^{eff} - T_{sea}). \qquad (6.2)$$

The effective air temperature is defined by the last equality in this equation. Given the field of the distribution of the short wave flux S and λ, the effective air temperature can easily be calculated. This last form is often used as it is a simple way to provide forcing for an ocean model - all that is needed is datasets of λ and the effective air temperature. Global datasets of λ are given by Oberhuber (1988). In our model the formulation is slightly different but is exactly equivalent to the approach of Haney as outlined above. We simply linearise the net surface heating around the climatological SST. So:

$$Q_{net}(T) = Q_{net}(T_c)+\lambda(T_c-T) \qquad (6.3)$$

where T_c is the climatological SST and T the model predicted SST. To see that this is equivalent to (2), (2) can be rewritten as:

$$Q_{net} = \lambda(T_{air}^{eff}-T) = \lambda(T_{air}^{eff}-T_c+T_c-T)=Q_{net}(T_c)+\lambda(T_c-T) \qquad (6.4)$$

where the fact that, by definition, has been used. It is the form (3) that we use in the model.

6.2 Important references

Haney, R.L., (1971) Surface thermal boundary condition for ocean circulation models. J. Phys. Oceanogr. 1, 241-248.

Oberhuber, J.M., (1988) The budgets of heat, buoyancy and turbulent kinetic energy at the surface of the global ocean, Max-Planck Institute Report No.15, MPI, Hamburg.

Philander, S.G.H. and A.D. Seigel, (1985) Simulation of El Nino of 1982-1983. In Coupled Ocean-Atmosphere Models (ed. J.C.J. Nihoul). Amsterdam: Elsevier.

6.3 Factors influencing choice of settings

It is clear from (3) that the value of the Haney coefficient λ essentially determines how far the modelled SSTs are allowed to drift from the climatological values. The smaller the value of λ the greater the drift in SST away from climatology. Oberhuber (1988) gives geographically and seasonally varying values of λ although there seems little justification in using such complexity in specifying this parameter in the model. A reasonable approach is to use an area average mean value appropriate to the ocean area under consideration.

6.4 Importance in model

For a model that is Haney forced the value of the Haney coefficient is crucial in determining the modelled SST. When using this forcing it is inappropriate to compare the modelled and climatological SST as a good deal of the solution in the model is input via this surface forcing.

6.5 Resolution dependence and recommended values

There is no resolution dependence in this parameter. As to the value of λ use the global fields of Oberhuber (1988) as a starting point.

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7: Parameters for the mixed layer model

7.1 Definitions and parameters

The mixed layer model used in the Unified Ocean Model is based on the simple ideas of Kraus and Turner (1967) and Gill and Turner (1976). In the highly turbulent region near to the ocean surface (the wind-wave zone) there is a considerable generation of turbulent kinetic energy (TKE) which can lead to vertical mixing in the ocean water column. This source of TKE has the dimensions of the work done by the wind on the ocean surface and can be scaled by the ocean friction velocity u_* as:

$$W = \lambda \rho_w u_*^3 \tag{7.1}$$

where ρ_w is the density of water and λ is a non-dimensional parameter of order unity. (See Davis et al (1981) for details). Now by definition:

$$\tau = \rho_w u_*^2 = \rho_a v_*^2 = \rho_a C_D V^2 \tag{7.2}$$

where τ is the magnitude of the surface stress at the air sea interface, ρ_a the density of air, v_* the atmospheric friction velocity, C_D the momentum drag coefficient and V the reference level wind speed. It follows from (1) and (2) that the wind mixing energy W is related to the wind V by:

$$W = \lambda(\rho_a C_D)^{3/2} V^3 \tag{7.3}$$

So for a given wind speed the coefficient determines the implied wind-mixing energy. In the code this parameter is called LAMDA.

W is the wind-mixing energy at the sea-surface and in the mixed layer model is used to do work against gravity in mixing down the near surface water of lower density. Now for shallow mixed layers (less than 30m say) it is reasonable to assume that W is also the mixing available at the base of the mixed layer. However, in practice the energy available for mixing at the mixed layer base will decrease as the mixed later depth increases (even though the

mixed layer, by definition, is of uniform density). This is taken into account in the model by introducing an exponential decay of windmixing energy with depth. If the model levels are labelled by j=1,2,3,.... and z_j are the depths of the model levels, then the windmixing energy available at depth z_n is given by:

$$W_n = (W_{n-1} - \Delta P_{n-1,n}) \exp(-z_n/\delta) \qquad (7.4)$$

where W_{n-1} is the energy available at the previous level, $P_{n-1,n}$ is the potential energy change in mixing levels n-1 and n and is a decay length scale. Essentially this means that the windmixing energy is not only used in mixing buoyant water downwards against gravity but there is also a 'background' dissipation of the TKE as represented by the exponential term in (4). The decay length scale is called DELTA in the code.

When there is a net surface cooling the model convectively overturns. These convective motions generate additional TKE and the buoyancy term in the TKE equation, that for surface heating is the major sink, is now a source term (see Kraus (1977) for details). It turns out (from analysis of the TKE equation) that the generation of TKE by the overturning convection is equal to the implied potential energy change in the water column. However, field experiments show that most of this convectively generated TKE is dissipated within the mixed layer and only a small fraction (typically 15%) is available for entrainment at the mixed layer base. It is likely that this fraction will itself be a function of mixed layer depth although these paremeters are only crudely known and there is little justification for introducing this complexity. In the model the fraction of convectively generated TKE used in further mixing is given by the dimensionless number EPSILON (ε).

7.3 Important references

Kraus, E.B. and J.S. Turner, (1967) A one-dimensional model of the seasonal thermocline: II. The general theory and its consequences. Tellus, 19, 98-106.

Kraus, E.B. ed (1977) Modelling and prediction of the upper layers of the ocean. Pergamon Press, Oxford, 325pp.

Gill, A.E. and J.S. Turner (1976) A comparison of seasonal thermocline models with observation. Deep Sea Res., 23, 391-401.

Davis, R.E., R. de Szoeke and P. Niiler (1981) Variability in the upper ocean during MILE. Part II. Modelling the mixed layer response. Deep-Sea Res., 28A, 1453-1475.

7.4 Factors influencing choice of settings

The parameters λ and δ need to be set as a pair. There are various estimates of these parameters from ocean modelling studies and laboratory experiments. One of the aims of the MILE experiment was to set some error bounds on the value of λ and it is therefore appropriate to stick to these values until better estimates are available. In the MILE model there was no decay of windmixing with depth and so λ and δ should be chosen together so that the total decay of windmixing is the same as that in the MILE model for mixed layer depths of the order simulated in MILE. For example, the smaller the decay length δ - which means there is a large dissipation with depth - the larger the value of λ that is needed.

7.5 Importance in model

The summer time mixed layer depth and SST are very sensitive to the value of λ and the winter mixed layer depth will become very large if the decay with depth (scaled by δ) is not included. Even with the decay of windmixing with depth included the mixed layer depths will still become too deep if most of the convectively generated mixing energy is not dissipated (by setting ε). To obtain a stable equilibium cycle in the modelled mixed layer depth both the parameters δ and ε must be set so as to decay mixing energy when the mixed layer is deep.

7.6 Resolution dependence

These parameters are not fundamentally resolution dependent.

7.7 Recommended values and range of acceptable values

Recommended values are λ=0.7, and δ=100m. Acceptable values of λ range from 0.5 to 1.0 although there seems little reason not to use values equivalent to those obtained during MILE (that is $\delta=\infty$ and λ=0.69). (Acceptable values of δ depend on the application in hand - e.g. simulating relatively shallow summer time mixed layer δ can be set very large (effectively infinite so there is no decay of depth)). A suitable range for δ is between 20m and 200m. The recommended value of ε is 0.15 with an estimated acceptable range of about 0.14 to 0.17 (Davis et al, 1981). The model is insensitive over this range of ε.

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8. Parameters for Fourier filtering

8.1 Definitions of parameters

JFT0 Reference row used to define minimum along-row scale to be retained - tracers.

JFU0 Reference row used to define minimum along-row scale to be retained - currents

JFRST First row in southern hemisphere to be filtered - tracers.

JFT1 Last row to be filtered in first batch (rows 2..JFT1 filtered) - tracer

JFU1 Last row to be filtered in first batch (rows 2..JFU1 filtered) - currents

JFT2 First row to be filtered in second batch (rows JFT2..JMTM1 filtered) - tracers

JFU2 First row to be filtered in second batch (rows JFU2..JMTM1 filtered) - currents.

8.2 Important references

Unified Model documentation paper 40: the Ocean Model.

K Bryan, S Manabe and RC Pacanowski, 1975, A global ocean-atmosphere climate model. Part II. The oceanic circulation. JPO, 5, 30-46

8.3 Factors influencing choice of settings

These parameters are used to control the results from subroutine FILTR. They are used to set the sizes of arrays in DERVSIZE, and to control loops in CLINIC and TRACER.

Fourier filtering is used in the model to allow longer time steps to be used than the strict stability criteria would permit. In a model that uses a latitude-longitude grid, the spacing of points along a row decreases as the pole is approached. For numerical stability (see Bryan et al, 1975) the time step must be limited by the maximum phase speed of systems in the model and the grid length.

Clearly we do not wish the model runs to be limited by the grid length near the pole. Fourier filtering is used to increase the effective grid length, but remove small scale variations in the model fields along a row.

Although the linear arguments give a guide to the time step that can be used, some experimentation is always needed to ensure numerical stability, and often some adjustment of the filtering parameters is necessary.

The purpose of the parameters is to specify the range of rows that are to be filtered. A reference row is given; this is used to calculate the number of wave components that should be retained at each row to give the same effective resolution as the grid spacing at the reference row.

It is normal to perform no Fourier filtering in the southern hemisphere because the flows in the pole-most rows there are so weak. Often this is achieved by an update to the code that removes even the residual calculations that would be done if a single row were to be filtered.

8.4 Resolution dependence

The parameters are given in terms of the row number. Increasing resolution usually means changing the rows referred to by the variables (it would be useful if real latitudes could be referenced!). Choosing the new row depends on the new grid spacing and on the time step. If you increase the time step you should expect to need to move the filtered region closer to the equator, as should if you decrease the along-row grid length. Fourier filtering assumes a uniform along-row grid point spacing; although the code will work with a variable grid, it is not clear what the physical impact if the filtering would be.

8.5 Recommended values

Global 2.5° x 3.75° model that includes rows over Antarctica:
JFT0=7, JFU0=7, JFRST=6, JFT1=7, JFU1=7, JFT2=67, JFU2=66

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Appendix A: The stability of Rossby and Shallow Water Waves

Rossby Waves

Consider the simple problem of the linearised barotropic quasi-geostrophic equations on an infinite beta plane:

$$\partial/\partial t \ \nabla^2\psi = -\beta\psi_x \tag{A.1}$$

Denote the timestep by n, the grid number along the x axis by m, and along the y axis by l and look for wave-like solutions growing without changing shape (the von Neumann method).

$$\psi^n_{m,l}=\gamma^n expi(\mu m\Delta x + \lambda l\Delta y) \tag{A.2}$$

For waves of this form and simple centred differencing

$$\psi_x \rightarrow \frac{1}{2\Delta x}(\psi_{m+1,l}-\psi_{m,l+1}) = \frac{i}{\Delta x}\sin\mu\Delta x \ \ \psi_{m,l} \equiv iL\psi_{m,l} \tag{A.3}$$

$$\nabla^2\psi \rightarrow \frac{2\psi_{m,l}}{\Delta x^2}(\cos\mu\Delta x + \cos\lambda\Delta y - 2) \equiv D\psi_{m,l} \tag{A.4}$$

Substituting these expressions in (A.1) the condition for solutions of the form (A.2) is:

$$(\gamma - 1/\gamma)D = -2i\beta L\Delta t \tag{A.5}$$

$$(\gamma + i\frac{\beta L\Delta t}{D})^2 = 1 - (\frac{\beta L\Delta t}{D})^2 \tag{A.6}$$

and the solutions are stable if

$$\Delta t \prec \frac{D}{\beta L} \tag{A.7}$$

This condition is strictest for long waves with $\lambda = 0$. Performing Taylor expansions of cos $\mu \Delta x$ and sin $\mu \Delta x$ the condition becomes
which is the requirement that the timestep be short enough to resolve the wave period.

$$\Delta t < \frac{\mu}{\beta} \qquad \text{(A.8)}$$

Shallow water waves

Killworth et al. (1984) present an analysis of the shallow water equations on an f-plane which we briefly reproduce here. This analysis is relevant to internal inertia-gravity waves (see Gill section 8.2). We will take the basic equations to be

$$\begin{aligned} r\,\delta_t \overline{u}^t - fv &= -\delta_x \overline{p}^y \\ r\,\delta_t \overline{v}^t + fu &= -\delta_y \overline{p}^x \\ \delta_t \overline{p}^t + c^2(\delta_x \overline{u}^y + \delta_y \overline{v}^x) &= 0 \end{aligned} \qquad \text{(A.9)}$$

These equations are formulated for a B grid (not a C grid); r is used to denote the ratio DTTS/DTUV to avoid confusion with the semi-implicit Coriolis coefficient α. The Coriolis acceleration will be taken to be semi-implicit. Substituting in waves of the form (A.2) with amplitudes u,v and p gives:

$$\begin{aligned} rAu + ifBv + \frac{2}{\Delta x}\sin m' \cos l'\, p &= 0 \\ -ifBu + rAv + \frac{2}{\Delta y}\cos m' \sin l'\, p &= 0 \\ \frac{2}{\Delta x}\sin m' \cos l'\, u + \frac{2}{\Delta y}\cos m' \sin l'\, v + Ap/c^2 &= 0 \end{aligned} \qquad \text{(A.10)}$$

where

$$\begin{aligned} m' &= \frac{m\Delta x}{2} \; ; \; l' = \frac{l\Delta y}{2} \\ iA &= \frac{(\gamma - 1/\gamma)}{2\Delta t} \; ; \; B = \alpha\gamma + \frac{(1-\alpha)}{\gamma} \end{aligned} \qquad \text{(A.11)}$$

Evaluation of the determinant gives the condition for a solution.

$$r^2A^2 - f^2B^2 - 4rc^2S = 0 \qquad \text{(A.12)}$$

where S, which depends on m' and l', lies between 0 and 1.

To find the stability conditions it is easiest to work in terms of θ where $\gamma = \exp i\theta$ and $T = \cos\theta$. From the definitions

$$A^2 = -(1-T^2)\Delta t^2 \quad B = \cos\theta + 2(\alpha - 0.5)\sin\theta \tag{A.13}$$

Equation (A.12) is unchanged when the signs of α-0.5 and of θ are reversed. Thus it appears that the condition $\alpha > 0.5$ cannot be very significant.

On setting α=0.5 one easily finds that (A.12) reduces to

$$(r^2+f^2\Delta t^2)T^2 = r^2 - 4rc^2\Delta t^2 S/\Delta x^2 \tag{A.14}$$

and the solutions are stable providing

$$r > (4c^2\Delta t^2) \;/\; (\Delta x^2) \tag{A.15}$$

as anticipated in equation (1.6).

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Appendix B: numerical analysis of diffusion equations

The simple 1D diffusion equation

when discretised using the leapfrog scheme with the diffusion term calculated on the preceding timestep (the retarded scheme), takes the form

$$\partial T/\partial t = \kappa\, \partial^2 T/\partial x^2 \qquad \textbf{(B.1)}$$

where n is the timestep and i is the 1D (x) coordinate.

$$\frac{T^{n+1}-T^{n-1}}{2\Delta t} = \kappa\{\frac{T_{i+1}-2T_i+T_{i-1}}{\Delta x^2}\} \qquad \textbf{(B.2)}$$

We look for solutions of the form

$$T_i^n = \gamma^n \exp i\lambda x \qquad \textbf{(B.3)}$$

and substitute (B.3) into (B.2) obtaining

$$\gamma^{n+1} = \gamma^{n-1}\{1 + \frac{4\kappa\ \Delta t\ (\cos\lambda\Delta x - 1)}{(\Delta x)^2}$$

$$= \gamma^{n-1}\{1 - \frac{8\kappa\Delta t}{(\Delta x)^2}\sin^2(\lambda\Delta x/2)\} \qquad \textbf{(B.4)}$$

For stability we need

$$-1 \le 1 - \frac{8\kappa\Delta t\ \sin^2(\lambda\Delta x/2)}{(\Delta x)^2} \le 1$$

The r.h.s. inequality is always satisfied, whilst the l.h.s. inequality yields

$$\Delta t \leq \frac{(\Delta x)^2}{4\kappa} \qquad \textbf{(B.5)}$$

Noise due to the isopycnal diffusion scheme

Consider the following simplified form of the isopycnal diffusion equation:

$$\partial T/\partial t = \kappa(\partial^2 T/\partial x^2 - \frac{\rho_x}{\rho_y} \partial^2 T/\partial z \partial x) \qquad \textbf{(B.6)}$$

Using the leapfrog scheme as in the Cox code again this is discretised as:

$$T_{i,k}^{n+1} - T_{i,k}^{n-1} = \frac{2\kappa\Delta t}{\Delta x^2}\{T_{i+1,k} - 2T_{i,k} + 2T_{i-1,k} - \frac{R}{4}(T_{i+1,k+1} - T_{i,,k+1} - T_{i+1,k} + T_{i,k})\}^{n-1} \qquad \textbf{(B.7)}$$

where

$$R = \frac{-\rho_x/\rho_z}{\Delta z/\Delta x} \qquad \textbf{(B.8)}$$

is the ratio of the slope of an isotherm to the steepest slope "represented" by the model. Substituting

$$T_{i,k}^{n} = \gamma^n \alpha^i \beta^k \qquad \textbf{(B.9)}$$

into (B.7) gives

$$\gamma^2 - 1 = \frac{2\kappa\Delta t}{\Delta x^2}\{ (\alpha - 2 + 1/\alpha) - R/4(\alpha\beta - \alpha/\beta - \beta/\alpha + 1/(\alpha\beta)) \} \qquad \textbf{(B.10)}$$

Let the term arising from the off-diagonal mixing be denoted by Γ:

$$\Gamma = R/4(\alpha\beta - \alpha/\beta - \beta/\alpha + 1/(\alpha\beta)) \qquad \textbf{(B.11)}$$

$\Gamma = 0$ if $\alpha = \pm 1$ or $\beta = \pm 1$; $\Gamma = R$ if $\alpha = -\beta = \pm i$; $\Gamma = -R$ if $\alpha = \beta = \pm i$. The mode (B.9) is unstable if the r.h.s. of (B.10) is positive. So off-diagonal mixing by itself is unstable for

chequer-board patterns in the x-z plane with wavelengths of 4 grid points in each direction. The phase of the growing waves depends on the sign of R (i.e. on the direction of slope of the isopycnals). For modes of this form (i.e. with $\Gamma = 1$) (B.10) gives

$$\gamma^2 - 1 = \frac{2\kappa\Delta t}{\Delta x^2}(|R| - 2) \qquad \textbf{(B.12)}$$

The modes are hence unstable (and have zero phase speed) when $|R| > 2$.

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